Models of Set Theory II - Winter 2015/2016

Prof. Dr. Peter Koepke, Regula Krapf — Problem sheet 4

Problem 15 (2 points)**.** Find a forcing notion $\mathbb{P}$ which decides the Continuum Hypothesis in the following way: There are $\mathbb{P}$-generic filters G and H such that $M[G] \models \mathsf{CH}$ and $M[H] \models \neg\mathsf{CH}$.

Problem 16 (8 points)**.** Let $\langle \langle \mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha \rangle \mid \alpha \leq \omega \rangle$ denote the finite support iteration of the sequence $\langle \langle \dot{\mathbb{Q}}_n, \dot{\leq}_n \rangle \mid n \in \omega \rangle$.

(a) Let $\kappa \geq 2$ be a cardinal in M. Suppose that for each $n \in \omega$,

$$\mathbb{1}_n \Vdash^M_{\mathbb{P}_n} \text{“}\dot{\mathbb{Q}}_n \text{ has an antichain of size } \kappa\text{”}.$$

Show that every $\mathbb{P}_\omega$-generic extension $M[G]$ contains a surjective function $f : \omega \to \kappa$ which is not in M.

(b) Conclude that if for each $n \in \omega$, $\mathbb{1}_n \Vdash^M_{\mathbb{P}_n}$ “$\dot{\mathbb{Q}}_n$ is atomless” and G is M-generic for $\mathbb{P}_\omega$ then $M[G]$ contains a Cohen real over M.

Problem 17 (4 points)**.** Let $\langle \langle \mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha \rangle \mid \alpha \leq \kappa \rangle$ denote the finite support iteration of the sequence $\langle \langle \dot{\mathbb{Q}}_\alpha, \dot{\leq}_\alpha \rangle \mid \alpha < \kappa \rangle$. Prove the following staetements:

(a) If κ is finite and $\mathbb{1}_n \Vdash^M_{\mathbb{P}_n}$ “$\dot{\mathbb{Q}}_n$ is σ-closed” for all $n < \kappa$ then $\mathbb{P}_\kappa$ is σ-closed.

(b) The analogue of (a) for κ infinite is false.

Problem 18 (6 points)**.** A *three-step iteration* is an iteration of the form $\langle \langle \mathbb{P}_n, \leq_n, \mathbb{1}_n \rangle \mid n \leq 2 \rangle$. Prove (very rigorously!) that a three-step iteration can be written as a two-step iteration in two different ways.

Please hand in your solutions on Monday, 30.11.2015 before the lecture.